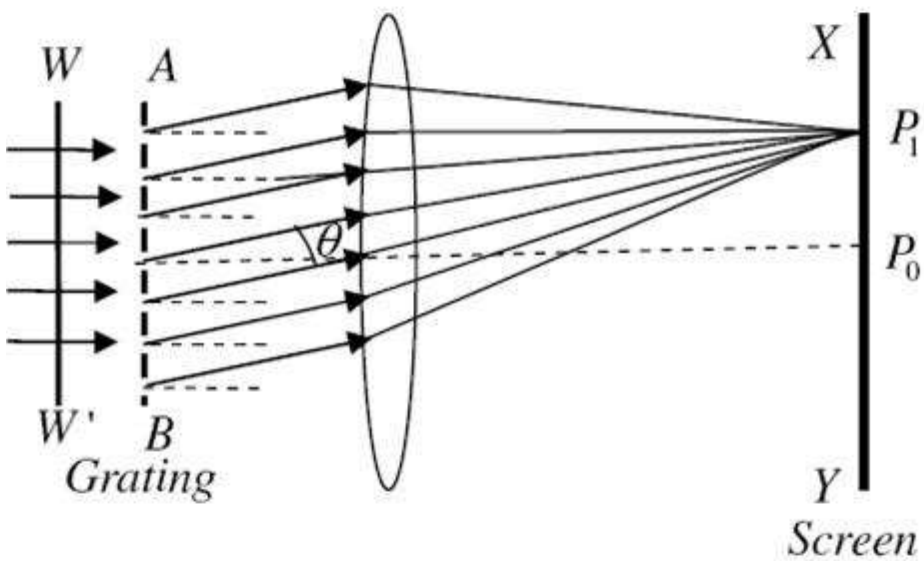
Diffraction due to a plane transmission grating:

An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as Diffraction grating.

Gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass, with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit. This is known as plane transmission grating. When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced.

A section of a *plane transmission grating AB* placed perpendicular to the plane of the paper is as shown in the figure.



Let *'e'* be the width of each slit and *'d'* the width of each opaque space. Then *(e+d)* is known as grating element and *XY* is the screen. Suppose a parallel beam of monochromatic light of wavelength 'λ' be incident normally on the grating. By Huygen's principle, each of the slit sends secondary wavelets in all directions. Now, the secondary wavelets travelling in the direction of incident light will focus at a point P_o on the screen. This point P_o will be a central maximum.

Now consider the secondary waves travelling in a direction inclined at an angle 'θ' with the incident light will reach point P_1 in different phases. As a result dark and bright bands on both sides of central maximum are obtained.

The intensity at point P_1 may be considered by applying the theory of Fraunhofer diffraction at a single slit. The wavelets proceeding from all points in a slit along their direction are equivalent to a single wave of amplitude $A\frac{sin\alpha}{\alpha}$ starting from the middle point of the slit, Where $\alpha = \frac{\pi}{\lambda}esin\theta$

If there are N slits, then we have N diffracted waves. The path difference between two consecutive slits is $(e+d)sin\theta$. Therefore, the phase difference

$$\delta = \frac{2\pi}{\lambda}(e+d)sin\theta = 2\beta \quad \text{........(1)}$$

Hence the intensity in a direction 'v ' can be found by finding the resultant amplitude of N vibrations each of amplitude $A\frac{sin\alpha}{\alpha}$ and a phase difference of '2β '

Since in the previous case $a = A\frac{sin\alpha}{\alpha}; n = N; \delta = 2\beta$

Substituting these in equation $R = a\frac{sin(n\delta/2)}{sin(\delta/2)}$

The resultant amplitude on screen at P_1 becomes

$$R = \left(A\frac{sin\alpha}{\alpha}\right)\frac{sinN\beta}{sin\beta} \quad \text{..........(2)}$$

Thus Intensity at P_1 will be

$$I^2 = R^2 = \left(A\frac{sin\alpha}{\alpha}\right)^2\frac{sin^2N\beta}{sin^2\beta} \quad \text{........(3)}$$

The factor $\left(A\frac{sin\alpha}{\alpha}\right)^2$

gives the distribution of Intensity due to a single slit while the factor

$$\frac{sin^2 N\beta}{sin^2\beta}$$

gives the distribution of intensity as a combined effect of all the slits.

Intensity Distribution:

Case (i): Principal maxima: The eqn (2) will take a maximum value if $sin\beta = 0$

$$\beta = \pm n\pi; n = 0, 1, 2, 3.....$$

$$\frac{\pi}{\lambda}(e + d)sin\theta = \pm n\pi$$

$$\boldsymbol{(e + d)sin\theta = \pm n\lambda} \text{.......(4)}$$

n = 0 corresponds to zero order maximum. For *n = 1,2,3,…* we obtain first, second, third,… principal maxima respectively. The ± sign indicates that there are two principal maxima of the same order lying on either side of zero order maximum.

Case (ii): Minima Positions: The eqn (2) takes minimum value if $sinN\beta = 0$ but $sin\beta \neq 0$

$$\therefore N\beta = \pm m\pi$$

$$N\frac{\pi}{\lambda}(e + d)sin\theta = \pm m\pi$$

$$N(e + d)sin\theta = \pm m\lambda \text{...........(5)}$$

Where ***m*** has all integral values except *m = 0, N, 2N, …, nN,* because for these values $sin\beta$ becomes zero and we get principal maxima. Thus, *m = 1, 2, 3, …, (N-1).* Hence

$N(e + d)sin\theta = \pm m\lambda$ where $m = 1, 2, 3....(N - 1), (N + 1), ...(2N - 1)....$

gives the minima positions which are adjacent to the principal maxima.

Case(iii): Secondary maxima: As there are *(N-1)* minima between two adjacent principal maxima there must be *(N-2)* other maxima between two principal maxima. These are known as secondary maxima. To find their positions

$$\frac{\mathrm{dI}}{\mathrm{d}\beta} = 0$$

$$\because \frac{sin\alpha}{\alpha} \neq 0; sinN\beta \neq 0$$

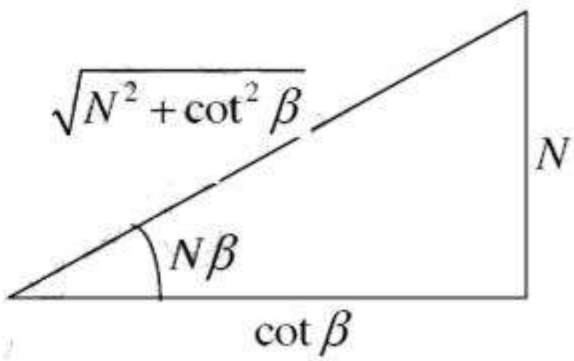
only

$$[NcosN\beta sin\beta - SinN\beta cos\beta] = 0$$

$\boldsymbol{Ntan\beta = tanN\beta}$............(6)

The roots of the above equation other than those for which $\beta = \pm n\pi$ give the positions of secondary maxima The eqn (6) can be written as

$$tanN\beta = \frac{N}{cot\beta}$$



From the triangle we have

$$sinN\beta = \frac{N}{\sqrt{N^2 + cot^2\beta}}$$

$$\therefore \frac{sin^2N\beta}{sin^2\beta} = \frac{N^2}{(N^2 + cot^2\beta)sin^2\beta}$$

$$\therefore \frac{sin^2N\beta}{sin^2\beta} = \frac{N^2}{N^2sin^2\beta + (1 - sin^2\beta)}$$

$$\therefore \frac{sin^2N\beta}{sin^2\beta} = \frac{N^2}{1 + (N^2 - 1)sin^2\beta}$$

$$I = I_o \left(\frac{sin\alpha}{\alpha}\right)^2 \frac{N^2}{1+(N^2-1)sin^2\beta}$$

Since intensity of principal maxima is proportional to N^2,

$$\frac{\text{Intensity of secondry maxima}}{\text{Intensity of Principal maxima}} = \frac{\frac{N^2}{1+(N^2-1)\sin^2\beta}}{N^2} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

Hence if the value of N is larger, then the secondary maxima will be weaker and becomes negligible when N becomes infinity.